Calculus

Name: Date:

Unit 3 Test 2 Review

Optimization

1. A farmer wants to fence in a rectangular pen so that is against a building. If he has 100 feet of fencing for the 3 sides of the pen, find the dimensions of the pen that will give him a *maximum area* in the pen.

2. Find the point on the graph of $y = (x + 2)^2$ that is closest to the point $(3,5)$.

3. A rectangular prism has a base that has a length that is twice the width. If the surface area of the box is 200 in^2, find the dimensions of the box that will *maximize* the volume.

One Dimensional Movement

4.

Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

(a) For $0 \le t \le 8$, when is particle P moving to the left?

(b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

(c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

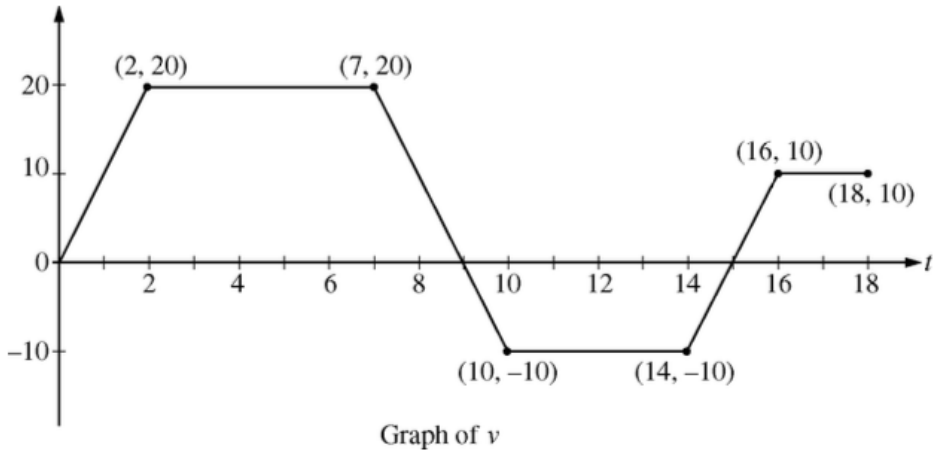
5.

For $t \ge 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

6.



Graph of v

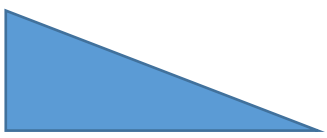
A squirrel starts at building A at time $t = 0$ and travels along a straight, horizontal wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

(b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A ?

Related Rates

7. The longer leg of a right triangle is shrinking at 2 inches per second, and the shorter leg of the triangle is stretching at 1 inch per second. Find the rate of change of the hypotenuse when the longer leg is 10 inches and the shorter leg is 7 inches.



8. The radius of r of a sphere is increasing at a rate of 3 inches per second. Find the change in the volume $V = \frac{4}{3}\pi r^3$ when the radius is 6 inches.

9. A man 6 feet tall walks at a rate of 3 feet per second away from a light that is 18 feet above the ground. When he is 12 feet from the base of the light, at what rate is the tip of the shadow moving?