Calculus

Name: Date:

3H: Related Rates

Why do those last seconds of a sand timer seem to go faster than the first seconds? Whenever I played a timed game with one of these timers I'd think I have plenty of time, then those sands of time would just vanish so quickly!

In the Beginning…

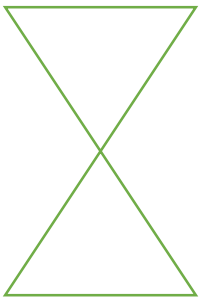
At the beginning of the timer, what about the shape of the sand changing and what is staying the same?

How about in those last sweet seconds?

Explore:

a) Find the rate of change of the volume of the sand timer we have in class.

b) Suppose that rate of Volume change is constant. Find the rate of change in height when $h = 1cm$. Do you need more info. about our timer?



c) Find the rate of change in height when $h = .5\ cm$.

d) Hmmm, is there a function that describes the change in height

GUIDELINES FOR SOLVING RELATED-RATE PROBLEMS

1. Identify all *given* quantities and quantities *to be determined*. Make a sketch and label the quantities.
2. Write an equation involving the variables whose rates of change either are given or are to be determined.
3. Using the Chain Rule, implicitly differentiate both sides of the equation *with respect to time t*.
4. *After* completing Step 3, substitute into the resulting equation all known values for the variables and their rates of change. Then solve for the required rate of change.

Let's take it down a dimension… to 2 dimensions:

Stretching Rectangle.
The length of a rectangle is decreasing by 2 inches per second and the width is increasing by 3 inches per second. When the length is 10 inches and width is 6 inches…

How fast is the perimeter changing?

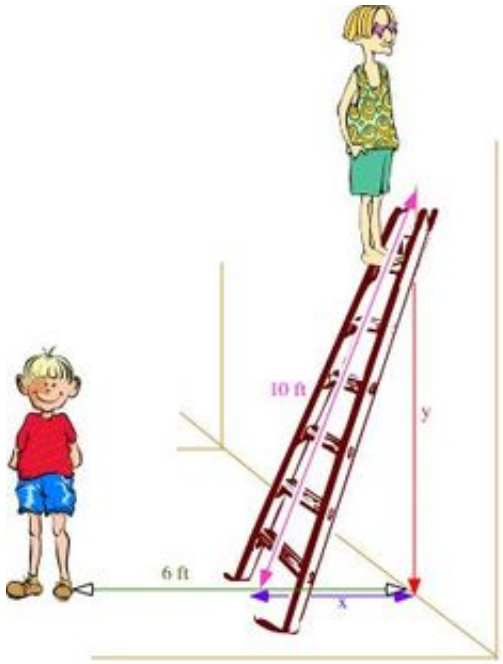
How fast is the Area changing?

Shrinking Sphere.

A spherical balloon is being inflated at a constant rate of 5 cubic inches per minute. When the radius of the balloon is 4 inches, how fast is the surface area of the balloon changing?

The Ladder Problem. (movement in two directions)

Suppose a 10 foot ladder is leaning against a wall and the base of it is being pulled away from the wall at a rate of 2 feet per second. When the base of the ladder is 6 feet away from the wall,



a) How fast is the ladder sliding *down* the wall?

b) How fast is the angle between the ground and the ladder changing?

c) How fast is the area of the triangle formed by the ladder, floor, and wall changing?

d) What is the acceleration of the ladder down the wall?

Cruising Cars

Two cars are driving toward each other on separate perpendicular roads. Romeo's car is 3 miles south of the intersection traveling at 40 mph, Juliette's care is 2 miles east of the intersection traveling at 50 mph. How fast is the distance between them change at this instant in time?

A Tough Ball Drop

A ball is dropped from a position that has a height of 50 feet, and is 8 feet away from a 30 foot lamppost. How fast will the shadow be moving 1.5 seconds after the ball is dropped?