Calculus

Name: Date:

3G: Particle Movement Problems

The Derivative as the "Rate of Change"

Okay, here is some crazy important stuff!

We know that a derivative at a point is...

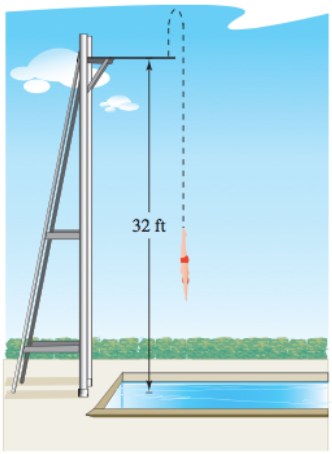
1. **The slope of the tangent line at that point**
2. **The instantaneous rate of change at that point**

If we let a function, $s(t)$, represent an object position with respect to time, then

- the "change in position" is called ***Velocity.***
  So, the objects velocity can be found by taking the derivative of the position $s(t)$!
- Wait there's more, the "change in velocity" is called ***Acceleration.***
  So, the derivative of the object's velocity $v(t)$ gives us acceleration!

Position	$s(t)$
Velocity	$v(t) = s'(t)$
Acceleration	$a(t) = v'(t) = s''(t)$

Let's try it



At time $t = 0$, a diver jumps from a platform diving board that is 32 feet above the water (see Figure 2.21). The position of the diver is given by

$$s(t) = -16t^2 + 16t + 32$$ Position function

where s is measured in feet and t is measured in seconds.

a) What is the diver's initial velocity?

b) What is the maximum height of diver?

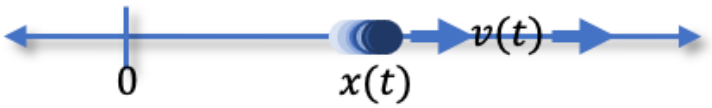
c) When does the diver hit the water?

d) What is the diver's velocity at impact?

e) What is the diver's acceleration at t = 1/2 seconds? 1 sec?

One-Dimensional Movement

The most simple of movements is called "one-dimensional movement" or "movement along a line". Physicists like to consider this motion because it allows us to focus on the basic characteristics of an object in motion.



- ***Displacement***: change in position on time interval (a, b) is $d = x(b) - x(a)$
- ***Velocity:*** $v(t) > 0 \rightarrow moving\ right \ ; \ v(t) < 0 \rightarrow moving\ left$
- ***Acceleration:*** $a(t) > 0 \rightarrow accelerating\ right \ ; \ a(t) < 0 \rightarrow accelerating\ left$
- ***Speed:*** The absolute value of Velocity (direction doesn't matter for speed)
  - ***Increasing speed:*** Velocity and Acceleration are in the ***same direction (i.e. same signs)***
  - ***Decreasing speed:*** Velocity and Acceleration are in ***different direction (i.e. different signs)***

Consider This:

A point is moving along the x axis for 10 seconds. It's position can be found using the function

$$x(t) = \frac{1}{3}x^3 - \frac{7}{2}x^2 + 10x + 10$$

a) What is the displacement for the first 6 seconds?

b) When (if ever) is the point stopped?

c) When is the point moving left?

When is it moving right?

d) What is the point's speed at $t = 3$?

e) Is the point speeding up or slowing down at $t = 3$?