Calculus Name: Date:

3D Exercises

Mean Value Theorem

Answer the following questions using the Mean Value Theorem (MVT). (This are all big AP Test questions)

_____ 3. Determine if the function $f(x)=x^3-x-1$ satisfies the hypothesis of the MVT on $[-1,2]$. If it does, find all possible values of c satisfying the conclusion of the MVT.

(A) $-\frac{1}{2}$

(B) $-1, 1$

(C) 0

$\boxed{\text{(D) 1}}$

(E) hypothesis not satisfied

$$\frac{\Delta y}{\Delta x} = \frac{f(2)-f(-1)}{2-(-1)} = \frac{5-(-1)}{3} = \frac{6}{3} = 2$$

$$f'(x) = 3x^2-1 = 2$$

$$3x^2 = 3$$

$$\sqrt{x^2} = \sqrt{1}$$

$$x = \pm 1$$

$x = 1$ is in $(-1,2)$

_____ 5. Which of the following functions below satisfy the hypothesis of the MVT?

✓ I. $f(x)=\frac{1}{x+1}$ on $[0,2]$ — $x \neq -1$

✓ II. $f(x)=x^{1/3}$ on $[0,1]$ — $x \in (-\infty, \infty)$

No III. $f(x)=|x|$ on $[-1,1]$ — Pointy @ $x=0$

(A) I only (B) I and II only (C) I and III only (D) II only (E) II and III only

_____ 6. As a graduation present, Jenna received a sports car which she drives very fast but very, very smoothly and safely. She always covers the 53 miles from her apartment in Austin, Texas to her parents' home in New Braunfels in less than 48 minutes. To slow her down, her dad decides to change the speed limit (he has connections.) Which one of the speed limits below is the highest speed her father can post, but still catch her speeding at some point on her trip?

(A) 55 mph (B) 70 mph (C) 65 mph (D) 50 mph (E) 60 mph

Avg Rate of change $\frac{53\text{mi}}{48\text{min}} \cdot \frac{60\text{min}}{1\text{hr}} = 66.25\text{mph}$

11. **(Calculator permitted)** For $f(x)=-x^4+4x^3+8x^2+5$ on $[0,5]$

(a) Determine if the MVT can be applied on the given interval. If so, find the value(s) guaranteed by the theorem.

f(x) is continuous & differentiable, so MVT applies

$$\frac{f(5)-f(0)}{5-0} = \frac{80-5}{5-0} = \frac{75}{5} = 15 \quad / \quad x=.674,\ x=3.794$$

(b) Find the equation of the secant line on $[0,5]$ Point $(5,80)$ or $(0,5)$

$$y-y_1=m(x-x_1)$$

(c) Find the equation of the tangent line at any value of c found above. $(.673, 9.647)$

$(3.79381, 131.403)$

10. Determine if the MVT can be applied to the following functions on the given interval. If so, find the exact value(s) guaranteed by the theorem. Be sure to show your set up in finding the value(s).

(a) $f(x)=\ln(x-1)$ on $[2,4]$

* Continuous & Differentiable on $(2,4)$. So MVT applies

$$\frac{\Delta y}{\Delta x} = \frac{f(4)-f(2)}{4-2} = \frac{\ln 3-\cancel{\ln 1}^{\,0}}{2} = \frac{\ln 3}{2}$$

$$f'(x) = \frac{1}{x-1} = \frac{\ln 3}{2}$$

$$2 = \ln 3(x-1)$$

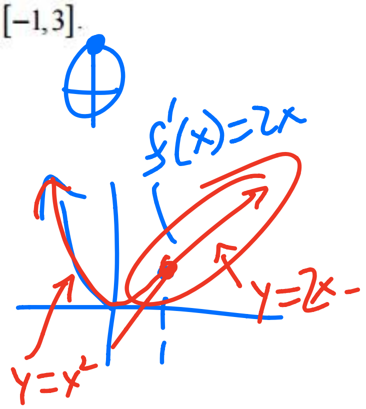
$$\boxed{\frac{2}{\ln 3}+1 = x}$$

(b) $f(x)=\begin{cases}\arcsin x, & -1\le x<1\\ \frac{x}{2}, & 1\le x\le 3\end{cases}$ on $[-1,3]$.

$$\lim_{x\to 1^-} f(x) = \arcsin(1) = \frac{\pi}{2}$$

$$\lim_{x\to 1^+} f(x) = \frac{1}{2}$$

Not Continuous @ $x=1$. MVT doesn't apply.



(c) $g(x)=\frac{x+1}{x}$ on $\left[\frac{1}{2},2\right]$

$x\neq 0$

g(x) is continuous & differentiable on $(\frac{1}{2},2)$. So, MVT applies

$$\frac{\Delta y}{\Delta x} = \frac{g(2)-g(\frac{1}{2})}{2-\frac{1}{2}} = \frac{\frac{3}{2}-3}{\frac{3}{2}} = \boxed{-1}$$

$$g'(x) = \frac{x-(x+1)}{x^2} = \frac{-1}{x^2} = -1$$

$x^2=1 \longrightarrow x=\pm 1$ $\boxed{\text{on } [\tfrac{1}{2},2]\ x=1 \to g(1)=-1}$

(d) $f(x)=2\sin x+\sin 2x$ on $[0,\pi]$

f(x) is continuous & differentiable on $[0,\pi]$

So MVT applies

$$\frac{\Delta y}{\Delta x} = \frac{f(\pi)-f(0)}{\pi-0} = \frac{0-0}{\pi} = 0$$

$$f'(x) = 2\cos x + 2\cos 2x$$

A Few Curve Sketching Review Problems from the AP test

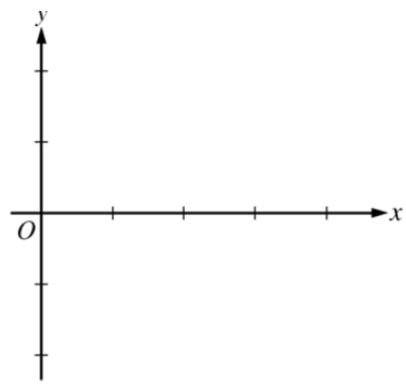
(AB4 2005)

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

4. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.



(c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function g defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

10. (AB3 1981) Let f be the function defined by $f(x) = 12x^{2/3} - 4x$

(a) Find the intervals on which f is increasing.

(b) Find the x- and y-coordinates of all relative maximum points. Justify.

(c) Find the x- and y-coordinates of all relative minimum points. Justify.

(d) Find the intervals on which f is concave downward.

(e) Using the information found in parts (a), (b), (c), and (d), sketch the graph of f.

12. Sketch the graph of a function that satisfies all of the following conditions.

(a) $f'(x) > 0$ for all $x \neq 1$, $\lim_{x \to 1^-} f(x) = \infty$, $\lim_{x \to 1^+} f(x) = -\infty$, $f''(x) > 0$ if $x < 1$ or $x > 3$, and $f''(x) < 0$ if $1 < x < 3$.

(b) $f'(x) > 0$ if $-2 < x < 2$, $f'(x) < 0$ if $x < -2$ and $x > 2$, $f'(2) = 0$, $\lim_{x \to \infty} f(x) = 1$, $f(-x) = -f(x)$, $f''(x) < 0$ if $0 < x < 3$, and $f''(x) > 0$ if $x > 3$