Calculus

Name: Date:

3C-2: Curve Sketching

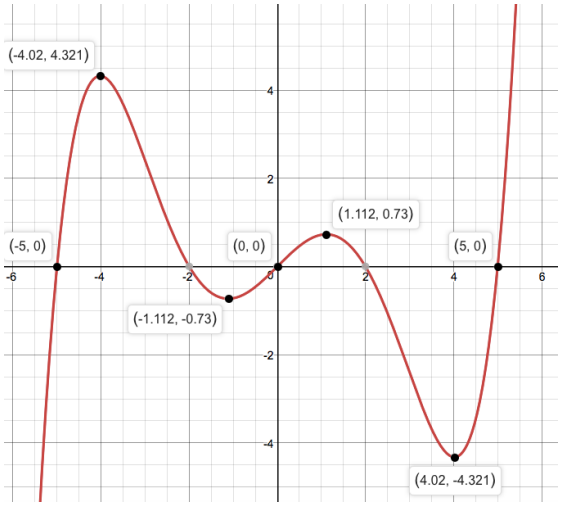
We have learned that $f'(x)$ can tell us where possible extrema are and give us intervals of increasing and decreasing, and $f''(x)$ can tell us about the concavity of the graph. Now we can put these together to analyze a function and understand how the function behaves.

Here is a summary of what we have learned about the 1st and 2nd derivative and their relationship to the graph.

Graphs of Derivative Functions

In the following problems, we will be provided with a graph of $f(x)$, $f'(x)$, or $f''(x)$. Use this information to answer the questions about the function.

1. If the graph above is the graph of a derivative f' on [-5.5,5.5],



   a) State the increasing intervals for $f(x)$. Justify.

   b) Where are the relative maximums and minimums? Justify.

   c) On what intervals is f concave up? Justify.

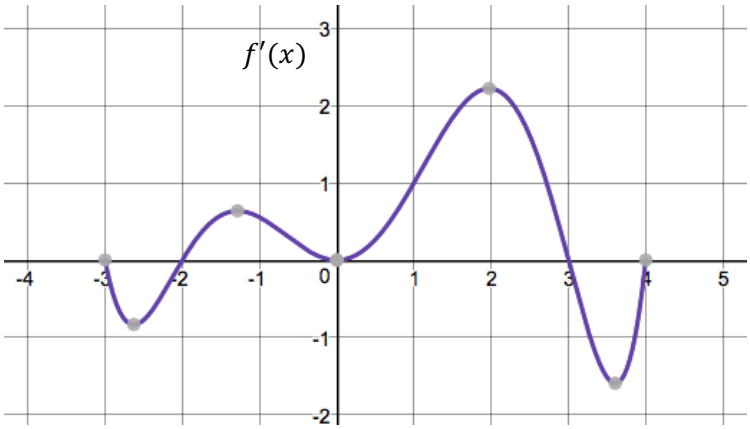
   d) At what approximate x-value(s) does $f(x)$ have an inflection point? Justify.

   e) Assuming that $f(0) = 0$, sketch a graph of f . If possible, determine the x-value(s) at which f attains its maximum and/or minimum values.

Summary: So, the information that we can get from a graph changes if the graph we have is the graph of the original function itself, or its 1st derivative, or its 2nd derivative. Here is a table that summarizes what clues we get from each type of graph.

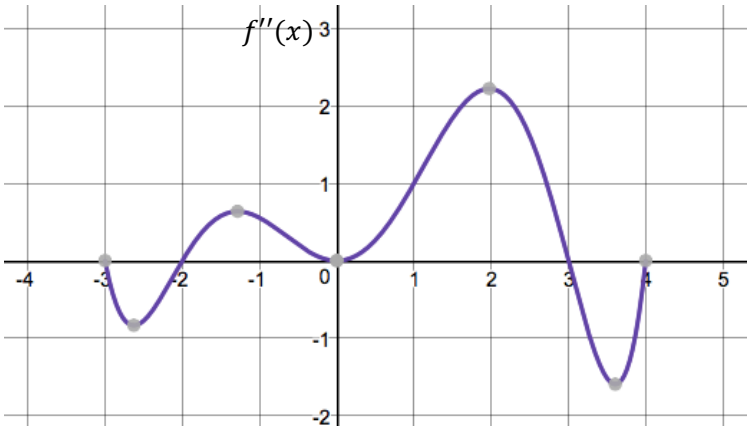
		0	Positive	Negative
f	y-values of the function	Root/zero of f ; x-intercept	Graph of f is above the x-axis	Graph of f is below the x-axis
f'	Slope: How the y-values are changing	Critical value of f; Possible Locations of relative max/min	Graph of f is increasing	Graph of f is decreasing
f''	Concavity: How the slopes are changing	Possible point of inflection;	Graph of f is concave up ; slopes of f are increasing	Graph of f is concave down ; slopes of f are decreasing

2. If the graph to the right is the **graph of $f'(x)$** on [-3,6] what important information can you find out about $f(x)$?



Assuming that $f(0) = 0$, sketch a possible graph of $f(x)$ on the same axes.

3. If the graph to the right is the **graph of $f''(x)$** on [-3,6] what important information can you find out about $f(x)$?



Assuming that $f(0) = 0$, sketch a possible graph of $f(x)$ on the same axes.

Curve Sketching from Equations

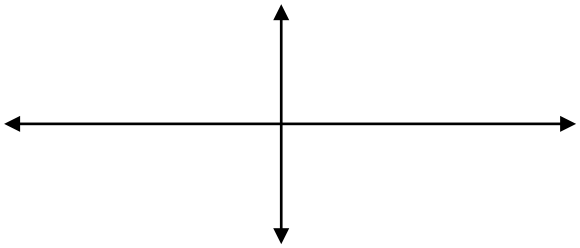
Now let's graph a few functions from their equations.

Steps for Curve Sketching:

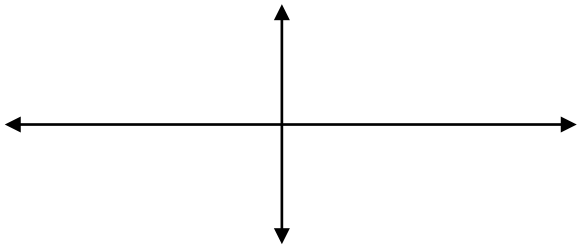
1. Pre-Calc stuff:
   Find the intercepts, asymptotes, end behavior, and symmetry of the graph.
2. Locate the x-values for which $f'(x)$ and $f''(x)$ either are zero or do not exist.
3. Use this to locate the extrema and inflection points.
4. Sketch the curve to "fill in the gaps".

Use the information provided by f, f', f'' to sketch a graph of the function $f(x)$ for each of the following:

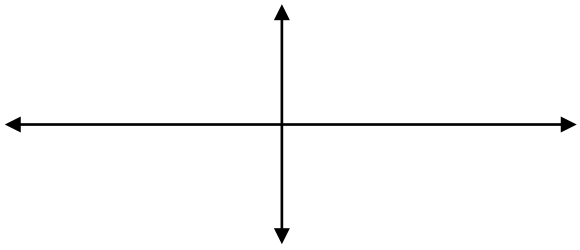
a) $f(x) = -x^5 + \frac{5}{2}x^4 + \frac{40}{3}x^3 + 5$



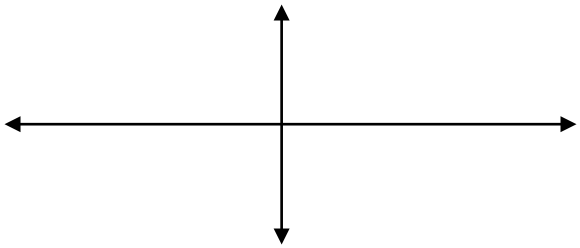
b) $f(x) = 2z^{5/3} - 5x^{4/3}$



c) $f(x) = \frac{x^2-2x+4}{x-2}$



d) $f(x) = \frac{x}{\sqrt{x^2+2}}$



e) $f(x) = e^{4-x^2}$