Name: Date:

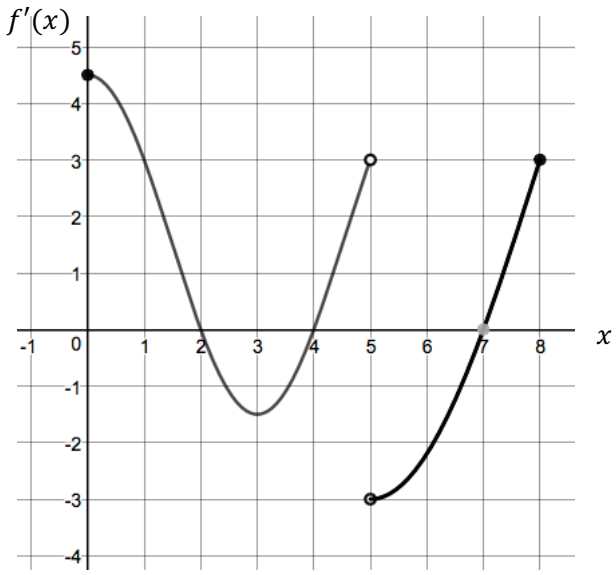
3C.2 Exercises

Curve Sketching

No calculator is allowed for these problems.

Part 1

1. The graph of the derivative f' of a continuous function $f(x)$ on [0,8] is shown to the right. Answer the following questions regarding the graph of f.



   a) On what open interval(s) is f increasing? Justify

   b) At what x-value(s) does $f(x)$ have a local maximum or minimum? Justify

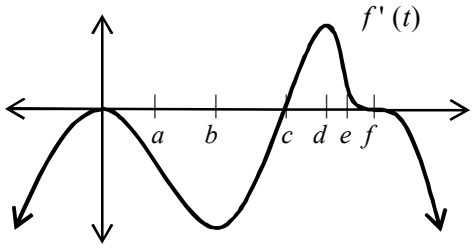
   c) At what intervals is it concave down?

   d) Where does f have an inflection point?

Assuming $f(0) = 0$, sketch a possible graph of f.
Determine the x-value(s) where f attains its maximum and/or minimum value(s).

No calculator is allowed for these problems.

Use the figure on the right to answer #1 to #8.



1. What are the critical values of $f(t)$?

   (A) b , d
   (B) 0 , b , d
   (C) 0 , b , d , f
   (D) 0 , c , f
   (E) 0 , c

2. When is $f(t)$ increasing?

   (A) (-∞ , 0) and (b , d)
   (B) (0 , b) and (d , +∞)
   (C) (-∞ , c) and (f , +∞)
   (D) (c , f)
   (E) (a , c) and (e , f)
   (F) (-∞ , a) , (c , e), and (f , +∞)

3. When is $f(t)$ decreasing?

   (A) (-∞ , 0) and (b , d)
   (B) (0 , b) and (d , +∞)
   (C) (-∞ , c) and (f , +∞)
   (D) (c , f)
   (E) (a , c) and (e , f)
   (F) (-∞ , a) , (c , e), and (f , +∞)

4. For each value of t below, classify $f(t)$ as a relative maximum, minimum, or neither.

   0 _____ a _____ b _____ c _____
   d _____ e _____ f _____

5. What are the possible points of inflection of $f(t)$?

   (A) 0 , a, c
   (B) a, c
   (C) 0 , b, d
   (D) b, d
   (E) 0
   (F) 0 , b, d, f

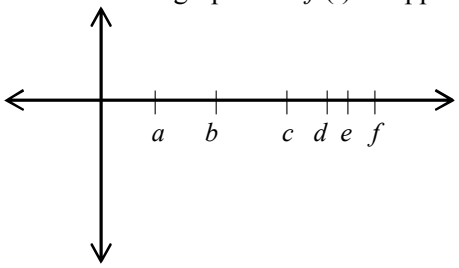
6. When is $f(t)$ concaving up?

   (A) (-∞ , 0) and (b , d)
   (B) (0 , b) and (d , +∞)
   (C) (-∞ , c) and (f , +∞)
   (D) (c , f)
   (E) (a , c) and (e , f)
   (F) (-∞ , a) , (c , e), and (f , +∞)

7. When is $f(t)$ concaving down?

   (A) (-∞ , 0) and (b , d)
   (B) (0 , b) and (d , +∞)
   (C) (-∞ , c) and (f , +∞)
   (D) (c , f)
   (E) (a , c) and (e , f)
   (F) (-∞ , a) , (c , e), and (f , +∞)

8. Sketch a graph of $f(t)$. Suppose $f(0) = 0$.



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9. On what interval is $f(x) = x^3 + x$ concave up?

   (A) ($-\infty$, $+\infty$)
   (B) (0 , $+\infty$)
   (C) ($-\infty$, 0)
   (D) (0 , 1)
   (E) (-1 , 0)

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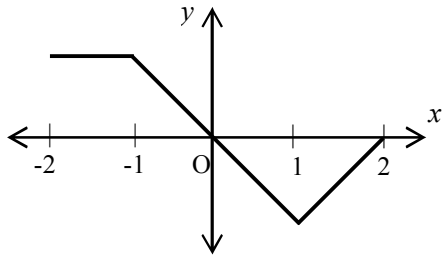
10. The absolute maximum of $f(x) = \dfrac{x}{x^2+1}$ is

    (A) 0
    (B) .25
    (C) .5
    (D) .75
    (E) 1

11. On what interval(s) is the graph of $f(x) = \dfrac{x}{x^2+1}$ concave down?

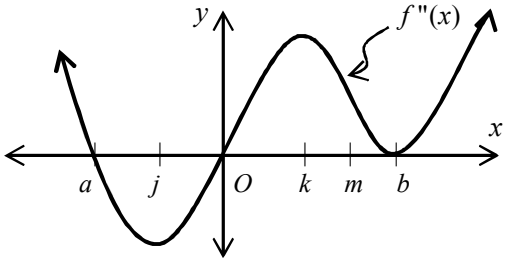
(A) $(0, \sqrt{3})$
(B) $(-\sqrt{3}, 0)$
(C) $(-\sqrt{3}, 0) \cup (0, +\infty)$
(D) $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$
(E) $(\sqrt{3}, +\infty)$



Graph of f'

12. The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f?

(A) f is decreasing for $-1 \le x \le 1$
(B) f is increasing for $-2 \le x \le 0$
(C) f is increasing for $-1 \le x \le 2$
(D) f has a local minimum at $x = 0$
(E) f is not differentiable at $x = -1$ and $x = 1$



13. The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?

(A) 0 and a only (B) 0 and m only (C) b and j only (D) 0, a, and b (E) b, j, and k

14. Over which interval(s) are the signs of both f' and f'' the same for $f(x) = 3x^4 - 4x^3 + 6$?

(A) $(0, \frac{2}{3})$
(B) $(-\infty, 0)$
(C) $(-\infty, 0) \cup (\frac{2}{3}, +\infty)$
(D) $(0, \frac{2}{3}) \cup (1, +\infty)$
(E) $(\frac{2}{3}, +\infty)$

15. Use the your work in #14 to sketch $f(x) = 3x^4 - 4x^3 + 6$.

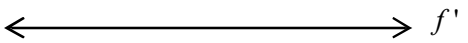
Classify the critical points for the following function as a relative maximum, relative minimum, or neither; determine intervals of concave up or down; then sketch the graph.

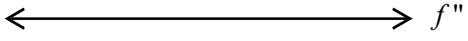
16. $f(x) = \dfrac{x}{x^2 - 1}$

relative maximum:

relative minimum:

neither:





$f(x)$ concaving up:

$f(x)$ concaving down:

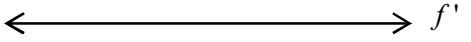
Classify the critical points for the following function as a relative maximum, relative minimum, or neither; determine intervals of concave up or down; then sketch the graph.

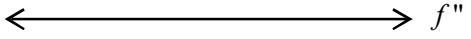
17. $f(x) = \frac{1}{3}x^3 - 2\ln|x|$

relative maximum:

relative minimum:

neither:





$f(x)$ concaving up:

$f(x)$ concaving down:

ANSWERS:							
1) D	3) C 5) F	6) A	8) graph 10) C	12) B	14) D	16) no max, no min, no neither ; conc up (-1,0) (1,+∞), conc down (-∞,-1) (0,1)
2) D	4) *c* min; *f* max	7) B	9) B 11) D	13) A	15) graph	17) no max, min at $\sqrt[3]{2}$, neither at 0 ; conc up (-1,0) (0,+∞), conc down (-∞,-1)