9-6: Graphing Quadratics using Properties

In this lesson, we will move from quadratic *equations* to quadratic *functions.* The key difference between these two is that a function has two variables, one input (*independent*) variable, and one output (*dependent*) variable. Our primary interest at this point is the graph of the quadratic function

$$f(x) = ax^2 + bx + c$$

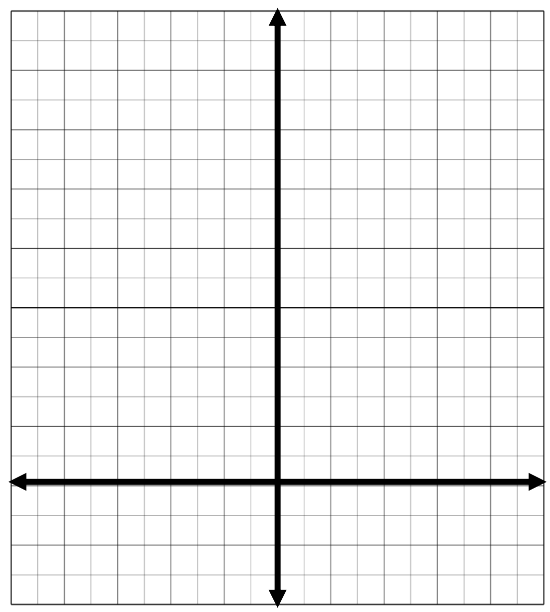
which will always be a U-shaped graph called a *parabola.*

The Graph of $f(x) = ax^2$

We begin with the simplest form of a quadratic function $f(x) = ax^2$.

Explore Complete the table of values for $f(x) = x^2$ and use it to graph the function.

x	$f(x) = x^2$
-3	
-2	
-1	
0	
1	
2	
3	



The shape of this graph is called a ***parabola***. The lowest point on the graph is called the ***vertex***. The vertical line that divides the parabola into two symmetrical halves is the ***axis of symmetry.***

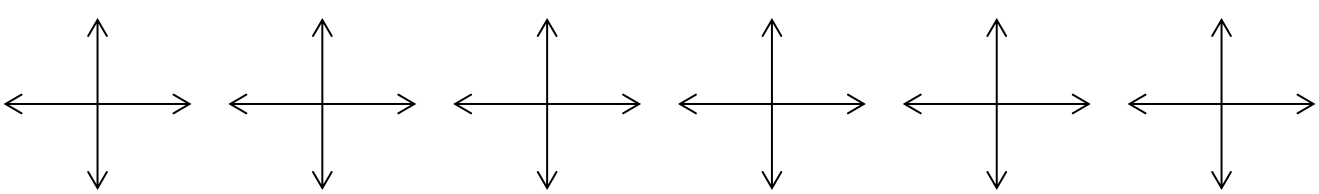
Consider this:

- What are the coordinates of the vertex of $f(x) = x^2$?
- What is the equation of the axis of symmetry for $f(x) = x^2$?

Explore more How does the graph of $f(x) = ax^2$ change as the value of a changes.

Use your graphing calculator to graph $y_1 = x^2$. Then graph each of the following in y_2 .

$y_2 = 3x^2$ $\quad$ $y_2 = \frac{1}{4}x^2$ $\quad$ $y_2 = \frac{1}{10}x^2$ $\quad$ $y_2 = -x^2$ $\quad$ $y = -3x^2$ $\quad$ $y = -\frac{1}{10}x^2$

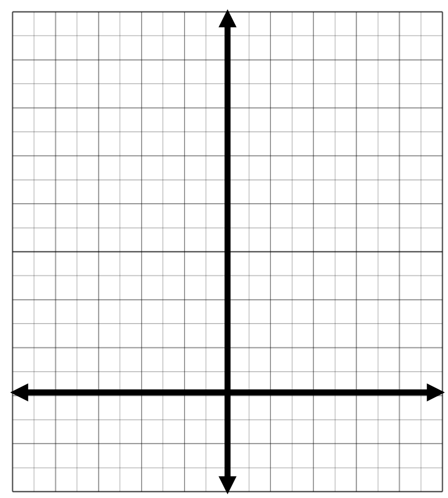


In the graph of $y = ax^2$, how does the value of a change the shape or position of the parabola relative to the graph of $y = x^2$?

- If $a > 0$, then the graph opens ________________________
- If $a < 0$, then the graph opens ________________________
- If $a < -1$, or $a > 1$, then the shape of the graph is ______________________________ or is “stretched” vertically.
- If $0 < a < 1$, or $-1 < a < 0$, then shape of the graph is ___________________________ or is “shrunk” vertically.

Example Use your observations to graph $y = \frac{1}{2}x^2$.

Describe how this graph is different than $y = x^2$.

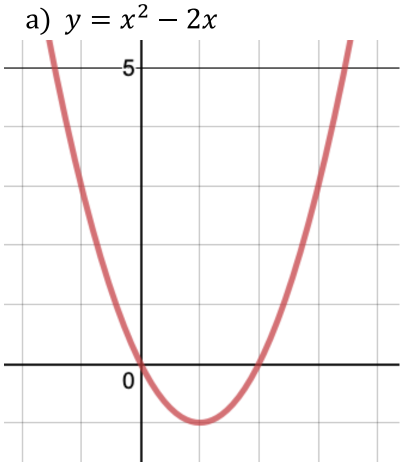


Finding the vertex from standard form

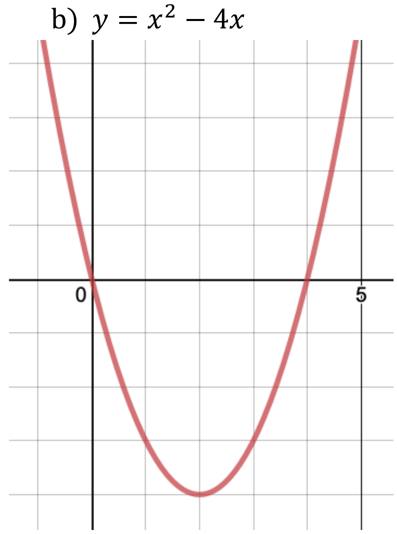
The **axis of symmetry** is the vertical line that cuts a parabola down the middle.

Find the equation of the axis of symmetry for these functions. How does the equation relate to the coefficients a and b in $y = ax^2 + b$?

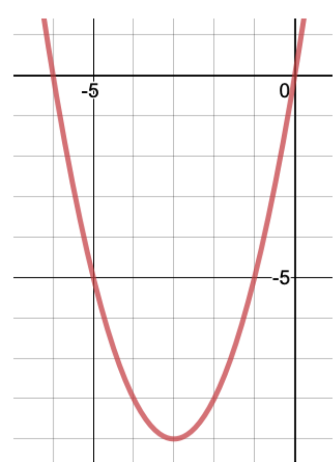
a) $y = x^2 - 2x$



b) $y = x^2 - 4x$



c) $y = x^2 + 6x$



So, we see that the axis of symmetry appears to be at $x = -\frac{1}{2} \cdot b$, which is close! When we change the a value in the function we will see that axis of symmetry falls at $x = -\frac{1}{2} \cdot \frac{b}{a}$ or $x = -\frac{b}{2a}$.

Finding the vertex in standard form: The general vertex form shows us that for any quadratic function in the form $f(x) = ax^2 + bx + c$ ➢ The x-coordinate of the vertex is at $-\frac{b}{2a}$, ➢ the **axis of symmetry** is at $x = -\frac{b}{2a}$, and ➢ the **vertex** is at $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ ➢ the maximum or minimum (depending on a) is at $f\left(-\frac{b}{2a}\right)$

Remember: *The axis of symmetry always passes through the vertex!*

Example. Find the axis of symmetry, the coordinates of the vertex, the minimum or maximum value, and sketch a *rough* graph for each function.

a) $f(x) = -2x^2 + 4x - 5$

b) $g(x) = 2x^2 + 2x - 15$

Determining the Intercepts

The intercepts of a function are key values that often need to be found. Since the intercepts are on the axes, we know that one of the coordinates must be zero. We find these intercepts as follows:

x-intercepts: when $y = f(x) = 0$, **y-intercept: when $x = 0$**

Example: Find the exact x-intercepts and y-intercepts of these functions.

a) $f(x) = x^2 + 3x - 10$

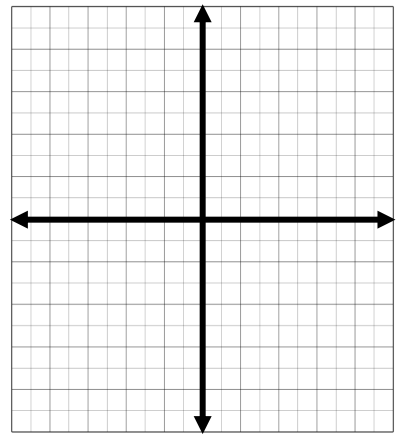
b) $g(x) = 2x^2 + x - 2$

We can now graph quadratic function in ***standard form*** ($y = ax^2 + bx + c$) by using these steps:

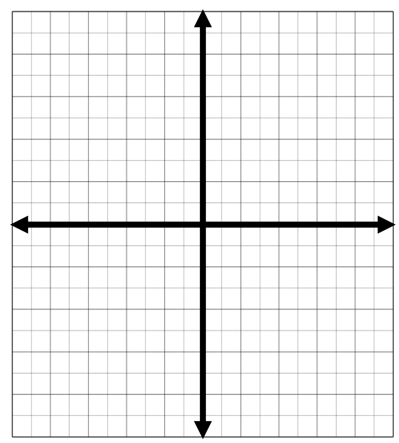
1. Find the coordinates of the vertex,
2. Find the x and y intercepts (if they are not too complicated),
3. Then use the a value to find several more points and graph. (Or plug a number in for x if needed)

Example Find the vertex and axis of symmetry of each function and sketch a graph.

a) $y = x^2 + 2x - 3$



b) $g(x) = 2\,x^2 - 8\,x + 9$



Example

The height of a certain ball that is thrown off of a 20 ft. building can be calculated using the function

$$h(t) = 20 + 10t - 16t^2$$

for the height $h(t)$, in feet, at time t, in seconds. Find the maximum height of the ball, then find the time when the ball will hit the ground.