<u>Take Home Quiz # 5</u>   2 pts. each

- Justify and show the means by which you arrive at your answers using equations, pictures, calculations, geometry, algebra steps, and/or technology. *You will not receive full credit if your answer is not supported by work that is legible and organized.*
- Place a box around your final answer. *It won't be graded if you do not do this!*
- Make your answers and their presentation in a professional and easily understandable format — make this your clearest and best work! *Points will be deducted for disorganized, sloppy work.*

12.1

1. Let $f(x) = 3x^2 + 4$ and $g(x) = 4x - 1$. Find the following

   a. $(f \circ g)(2) = f(g(2))$
   $$= 3(7)^2 + 4$$
   $$= \boxed{151}$$

   $g(2) = 4(2) - 1$
   $$= 7$$

   b. $(g \circ f)(2) = g(f(2))$
   $$= g(16)$$
   $$= 4(16) - 1 = \boxed{63}$$

   $f(2) = 3(2)^2 + 4$
   $$= 16$$

2. Let $f(x) = 3x^2 + 4$ and $g(x) = 4x - 1$. Find the following

   a. $(f \circ g)(x) = f(g(x))$
   $$= 3(4x-1)^2 + 4$$
   $$= 3(16x^2 - 8x + 1) + 4$$
   $$= 48x^2 - 24x + 3 + 4 = \boxed{48x^2 - 24x + 7}$$

   b. $(g \circ f)(x) = g(f(x))$
   $$= 4(3x^2 + 4) - 1$$
   $$= 12x^2 + 16 - 1 = \boxed{12x^2 + 15}$$

3. Consider the function $g(x) = (x+7)^3$

   a. Determine if the function is one-to-one

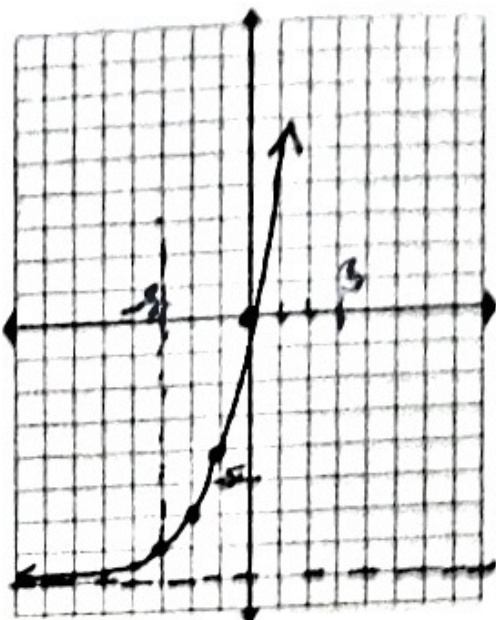   Using the Horizontal line test, we see that $g(x) = (x+7)^3$ is one-to-one because it passes the test!

   b. Find a formula for the inverse $g^{-1}(x)$.

   $$y = (x+7)^3 \longrightarrow x = (y+7)^3$$
   $$\sqrt[3]{x} = y + 7$$
   $$\sqrt[3]{x} - 7 = y$$

   $$\boxed{g^{-1}(x) = \sqrt[3]{x} - 7}$$

12.2

4. Graph $y = 2^{x+3} - 8$

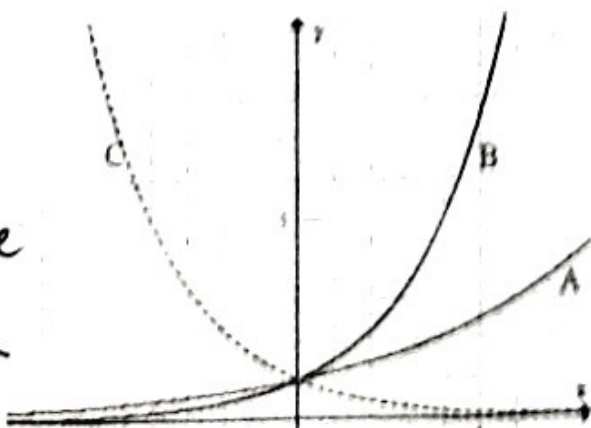5. The graphs to the right represent the functions

$$A(x) = a^x$$
$$B(x) = b^x$$
$$C(x) = c^x$$

List the bases a, b, and c from least to greatest. Explain how why they must be in this order.

$$c < a < b$$

We know $c < 1$ because it is exp. decay. The other two is exp. growth so $a > 1$ and $b > 1$.

Since $B(x)$ is steeper than $A(x)$, $b > a$.

6. The bacteria *Escherichia coli* are commonly found in the human bladder. Suppose that 3000 of the bacteria are preset at the time $t = 0$. Then t minutes later, the number of bacteria present can be approximated by

$$P(t) = 3000(2)^{t/20}$$

a. How many bacteria will be present after 10 minutes? $P(10) = 3000(2)^{10/20} = 4242.6$
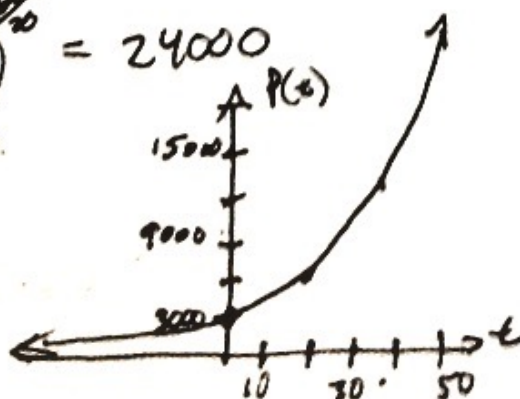
20 min? $P(20) = 3000(2)^{20/20} = 6000$

30 min? $P(30) = 3000(2)^{30/20} = 8485.28$

40 min? $P(40) = 3000(2)^{40/20} = 12000$

60 min? $P(60) = 3000(2)^{60/20} = 24000$

b. Sketch a *rough* graph of this function.

7. Show evidence that you can evaluate $\log_6 36$ without a calculator.

$$\log_6 36 = X$$
$$36 = 6^X$$
$$6^2 = 6^X$$
$$\boxed{X = 2}$$

8. Show evidence that you can evaluate $\log 1000$ without a calculator

$$\log 1000 = X$$
$$1000 = 10^X$$
$$10^3 = 10^X$$
$$\boxed{X = 3}$$

9. Write the equation in exponential form then solve for x $\log_4(x + 24) = 3$

$$X + 24 = 4^3$$
$$X + 24 = 64$$
$$\boxed{X = 40}$$

10. The function $F(d) = 184 - 16Ln(d - 3)$ models the flow (F) of a river (measured in cubic feet per second) as a function of the day (d). Find the flow of the river on day 36.

$$F(36) = 184 - 16\ln(36-3)$$
$$= 184 - 16\ln(33)$$
$$\approx 128.06$$

on day 36, the flow should be $\boxed{128 \text{ ft}^3/\text{sec}}$